Left Neighbor: _____   Right Neighbor: _____

Name: _____   Student ID: _____

Section TA: _____

This is a closed book quiz

1. **(2 points)** Show that $\displaystyle\sum_{i=k_n+1}^{4k_n} \frac{1}{i} \geq 1$. If you can't do that, for 1 point, show that

$$\sum_{i=k_n+1}^{4k_n} \frac{1}{i} \geq \frac{3}{4}.$$

2. **(4 points)** Prove that $\forall n \exists k_n \in$ the natural numbers, $\displaystyle\sum_{i=1}^{k_n} \frac{1}{i} \geq n$ by induction. You can use the result above, *whether or not you successfully proved it*, and you can assume that $k_{n+1} = 4k_n$. Make sure you give the Basis step (1 point), the Inductive Hypothesis (1 point) and the Inductive Step (2 points).

3. **(5 points)** Prove that $F(n) < 2^n$ by induction on n (where $F(n)$ is the n'th Fibonacci number, $F(0) = 0$ and $F(1) = 1$).