

let  $P(x) \rightarrow$  all even numbers

$Q(x) \rightarrow$  all odd numbers.

All numbers are integers.

1. (4 points) Show that  $\forall x P(x) \vee \forall x Q(x)$  and  $\forall x(P(x) \vee Q(x))$  are not logically equivalent.

$\forall x P(x) \rightarrow$  all numbers are even numbers  $\rightarrow$  False.

$\forall x Q(x) \rightarrow$  all numbers are odd numbers  $\rightarrow$  False.

$\forall x P(x) \vee \forall x Q(x) \rightarrow$  False

$\forall x(P(x) \vee Q(x)) \rightarrow$

True

2. (2 points) Express the negation of these statements so that all negation symbols immediately precede predicates:  $\exists x \exists y P(x, y) \wedge \forall x \forall y Q(x, y)$

$$\forall x \forall y \neg P(x, y) \vee \exists x \exists y \neg Q(x, y)$$

3. (4 points) Determine the truth value of each of these statements if the domain for all variables consists of all integers:

(a)  $\forall x \exists y(x^2 < y)$  True

x	$\frac{y}{=}$
2	5
10	101

(b)  $\exists x \forall y(x^2 < y)$  False

There is no x that satisfies the above statement

(c)  $\forall x \exists y(x + y = 0)$  True

$x = -y$	$\frac{y}{=}$	$\frac{y}{=}$
1	-1	-1
2	-2	-2

(d)  $\exists x \forall y(xy = y)$  True

$x = 1$  satisfies the above statement