Quiz 6: Cinco de Mayo, 2016

Left Neighbor: _____ Right Neighbor: _____

Name: _____ Student ID: _____

Section TA: _____

This is a closed book quiz

1. **(5 points)** Prove the below by induction for all natural numbers, $n \geq 7$. You get 1 point for stating the Inductive Hypothesis, 1 point for the Basis Step, and 3 points for the Inductive step.

$$3^n < n!$$

$P(7)$: $3^7 < 7!$

IH: $P(K)$: $3^K < K!$

IC: $P(K+1)$: $3^{K+1} < (K+1)!$ (To prove)

$3^K < K!$ [IH]

$3^K(K+1) < (K+1)!$ [multiply both sides by $K+1$]

$3^{K+1} < (K+1)!$ [$\because (K+1) > 3$]

2. **(5 points)** Prove the below by induction for all positive integers, n. You get 1 point for stating the Inductive Hypothesis, 1 point for the Basis Step, and 3 points for the Inductive step.

$$\sum_{i=1}^{n} i2^i = (n-1)2^{n+1} + 2$$

$P(1)$: $2 = 2$

IH: $P(K)$: $\sum_{i=1}^{K} i\,2^i = (K-1)\,2^{K+1} + 2$

IC: $P(K+1)$: $\sum_{i=1}^{K+1} i\,2^i = K \cdot 2^{K+2} + 2$

$$\text{LHS} = \sum_{i=1}^{K} i\,2^i + (K+1)2^{K+1} = (K-1)2^{K+1} + 2 + (K+1)2^{K+1}$$

$$= 2K \cdot 2^{K+1} + 2 = 2^{K+2} \cdot K + 2$$